

## Binomial Expansion - Coefficients



### Exercise 8D

1 Find the coefficient of  $x^3$  in the binomial expansion of:

a  $(3 + x)^5$

b  $(1 + 2x)^5$

c  $(1 - x)^6$

d  $(3x + 2)^5$

e  $(1 + x)^{10}$

f  $(3 - 2x)^6$

g  $(1 + x)^{20}$

h  $(4 - 3x)^7$

i  $(1 - \frac{1}{2}x)^6$

j  $(3 + \frac{1}{2}x)^7$

k  $(2 - \frac{1}{2}x)^8$

l  $(5 + \frac{1}{4}x)^5$

- (P) 2 The coefficient of  $x^2$  in the expansion of  $(2 + ax)^6$  is 60. Find two possible values of the constant  $a$ .

#### Problem-solving

$a = 2$ ,  $b = ax$ ,  $n = 6$ . Use brackets when you substitute  $ax$ .

- (P) 3 The coefficient of  $x^3$  in the expansion of  $(3 + bx)^5$  is  $-720$ . Find the value of the constant  $b$ .

- (P) 4 The coefficient of  $x^3$  in the expansion of  $(2 + x)(3 - ax)^4$  is 30. Find the three possible values of the constant  $a$ .

- (E/P) 5 When  $(1 - 2x)^p$  is expanded, the coefficient of  $x^2$  is 40. Given that  $p > 0$ , use this information to find:

a the value of the constant  $p$  (6 marks)

b the coefficient of  $x$  (1 mark)

c the coefficient of  $x^3$  (2 marks)

#### Problem-solving

You will need to use the definition of  $\binom{n}{r}$  to find an expression for  $\binom{p}{2}$ .

- (E/P) 6 a Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of  $(5 + px)^{30}$ , where  $p$  is a non-zero constant. (2 marks)
- b Given that in this expansion the coefficient of  $x^2$  is 29 times the coefficient of  $x$  work out the value of  $p$ . (4 marks)

- (E/P) 7 a** Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + qx)^{10}$ , where  $q$  is a non-zero constant. **(2 marks)**
- b** Given that in the expansion of  $(1 + qx)^{10}$  the coefficient of  $x^3$  is 108 times the coefficient of  $x$ , work out the value of  $q$ . **(4 marks)**
- (E/P) 8 a** Find the first three terms, in ascending powers of  $x$  of the binomial expansion of  $(1 + px)^{11}$ , where  $p$  is a constant. **(2 marks)**
- b** The first 3 terms in the same expansion are 1,  $77x$  and  $qx^2$ , where  $q$  is a constant. Find the value of  $p$  and the value of  $q$ . **(4 marks)**
- (E/P) 9 a** Write down the first three terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + px)^{15}$ , where  $p$  is a non-zero constant. **(2 marks)**
- b** Given that, in the expansion of  $(1 + px)^{15}$ , the coefficient of  $x$  is  $(-q)$  and the coefficient of  $x^2$  is  $5q$ , find the value of  $p$  and the value of  $q$ . **(4 marks)**
- (E/P) 10** In the binomial expansion of  $(1 + x)^{30}$ , the coefficients of  $x^9$  and  $x^{10}$  are  $p$  and  $q$  respectively. Find the value of  $\frac{q}{p}$ . **(4 marks)**

### Challenge

Find the coefficient of  $x^6$  in the binomial expansion of: **a**  $(3 - 2x^2)^9$  **b**  $\left(\frac{5}{x} + x^2\right)^8$